**Test 6 2016**

**SOLUTION Calculator Section**

**Time limit = 40 minutes.**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Score: \_\_\_\_\_\_\_\_\_\_ / 39**

**Calculators allowed**

**Access to approved Sample Mathematics Specialist formulae sheet is permitted.**

**Answers should be in 4 decimal places.**

**Question 1 (1,2,2,2,3) (10 marks)**

The burn time, T seconds, of a randomly chosen match produced by the Ever-Flame company is normally distributed, with a mean of 15 seconds and a standard deviation of 2.3 seconds.

1. Calculate P (T› 20)

**T ~ N (15 , 2.32 ) P ( T > 20 ) = 0.0149**

1. Find the value of k, given that 90% of the matches burn for longer than k seconds.

**P ( T > K ) = 0.90 K = 12.0524 K = 12 seconds**

1. If 10 matches are burned, find the probability that at least half burn for less than k seconds.

**X ~ Bin ( 10 , 0.1 ) P ( x ≥ 5 ) = 0.0016**

1. Every week the company tests its matches by measuring the burn times of 1500 randomly-chosen matches.

What is the probability that the average burn time of the matches in such a sample will be between 14.90 and 15.10 seconds?

Let X denote the average burn time from a random sample of 1000 matches

 **~ N ( 15 , 2.3/√1500 )2 P ( 14.90 ≤**   **≤ 15.10 ) = 0.9078**

1. The rival Sure-Fire company produces matches whose burn times have the same standard deviation as the Ever-Flame matches, but whose mean, µ seconds, is unknown. Scientists plan to estimate µ using the average burn time of matches in a random sample of Sure-Fire matches.

How large should this sample be, if the scientists are to be 95% confident that this estimate will be correct to within 0.1 seconds?

The half width of the 95% confidence interval is **1.96 (2.3 / √ n ) = 4.5 / √ n**

**If 4.5 / √n < 0.1 n > 2025** Sample size should be at least 2025**Question 2 (1, 1, 2, 3, 4, 2) (13 marks)**

The length of barramundi is approximately normally distributed with a mean of 320mm and a standard deviation of 100mm. For game fishing, a barramundi must be between 200mm and 500mm long to be considered of legal size.

1. What is the probability that a randomly caught barramundi is of legal size?

**X ~ N ( 320 , 100 )2 P ( 200 < X < 500 ) = 0.85**

1. A fisherman catches 100 barramundi in a week. What is the expected number of legal sized fish in his catch?

**N = 100 x 0.85 = 127.5** ~ **128**

1. What is the probability that a legal-sized barramundi is over 450mm in length?

**P ( X > 450 | 200 < X < 500 ) = 0.0609 / 0.85 = 0.072 ~ 0.07**

1. Calculate the interquartile range of the barramundi population.

**P ( X < Q3 ) = 0.75 Q3 = 387.55**

**P ( X < Q1 ) = 0.25 Q1 = 252.55 IQR = 134.9 (1 dp)**

1. A fisheries researcher suspects that the length of the barramundi population may have changed over time. She intends to investigate this by taking random samples of barramundi and calculating the mean length. Assume that the standard deviation of the fish population is still 100 mm.
2. Her first sample of 40 barramundi had a mean length of 265mm. Use this to calculate a 90% confidence interval for the mean length of the population, and explain whether this provides strong evidence that the population mean had changed from 320mm.

 **~ N (320, (100 / (√40))2 )**

A 90% confidence interval is **320 ± 1.645 x (100/√40) = [3226 , 3173]**

As 256 belongs to this interval, the sample does not provide strong evidence that the

length of barramundi has changed.

1. With her second sample, she wants to obtain a 95% confidence interval for the mean length of the barramundi population which has a width of no more than 30mm. What sample size should she select?

No more than 30mm 15 mm either side of the mean.

**15 = 1.96 x ( 100 / √ n) n = 170.74 ~ 170**

**Question 3 (1, 3, 1, 2, 3, 1, 3, 2) (16 marks)**

A study found that 60 per cent of people exhibiting common influenza symptoms recovered without taking any medication. A random sample of 35 people who had developed influenza symptoms was taken.

Let X denote the number of people in this sample who recovered without taking any medication.

1. Is X discrete or continuous?

Discrete

1. State the probability distribution of X and the mean and standard deviation of the distribution.

X ~ B (35, o.6)

µ = 21 and Ó = √ 4.8 ≈ 2.898

1. What is the probability, correct to three decimal places, that
2. exactly 20 people recovered without any medication?

P ( X = 20 ) = 0.128

1. at least 18 but no more than 25 people recover without any medication?

P ( 18 ≤ X ≤ 25 ) = 0.828

1. Trial groups of 30 people from each of 15 different suburbs were then surveyed. Let  denote the mean number of people per trial group who recover without any medication.
2. State the probability distribution of  and the mean and standard deviation of this distribution.

Since n = 15 and the background population is Binomial, the Central Limit Theorem cannot be applied with any confidence. However we can say that the distribution of  approaches the normal distribution given by  ~ N (µß , Ó2ß )

Where µß = µx = 24 and Óß = 2.1909 / (√15) = 0.5657

 ~ N (24, 0.32)

1. Determine P ( ≥ 25 )

P ( ≥ 25 ) = 0.03855

1. Determine a 95% confidence interval, to three decimal places, for the population mean number of people per trial who recover without medication.

1.96 x 0.5657 = 1.109 confidence interval is [ 24 ± 1.109] = [22.891, 25.109]

1. The researcher who conducted the trials in the 15 suburbs calculated a mean of 25 people who recovered without medication per trial. The researcher concluded that a smaller percentage of influenza suffers take medication than had been assumed. Does this mean support her conclusion? Explain.

No, as the mean value 25 is within the 95% confidence interval calculated in (iii) above.

Alternatively, using 25 as the sample mean, a confidence interval for the population mean would be [23.891, 26.109]. This interval contains the value of 24 which was assumed to be the population mean. Since n p = µ, and 95% on confidence intervals about the sampling distribution mean should contain µx , the researcher would be incorrect to suspect that p

Should be greater than 0.8

END OF PAPER